

# Quantity and quality effects of advertising: a demand system approach

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## Abstract

Quantity and quality effects of advertising are investigated in this article. A censored demand system is estimated for fish, beef, pork, and other meats using Norwegian household data. In the demand model, generic fish advertising and meat advertising is introduced as demand shifters, and at the same time unit values are treated as endogenous. The unit values capture the quality of the commodities. Empirical findings show that advertising can have both quantity and quality effects on household demand.

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## 1. Introduction

The effects of generic advertising on consumer demand for food commodities have been examined extensively over the past two decades (see Ferrero et al., 1996, for an annotated bibliography of research). All previous studies have focused exclusively on the issue of whether advertising increases the quantity of food purchased. Food commodities are, in general, aggregated from many heterogeneous products. For example, beef comprises loin, roast, steak, ground, chuck, and so on. Consequently, in addition to potentially increasing the total quantity of meat purchases, meat advertising may also change the composition of the household's meat purchases. The purchase composition usually determines aggregate commodity quality. Therefore, advertising may have quality as well as quantity effects on consumer purchases. Information on both of these effects can lead to a better understanding of how consumers respond to advertising.

In order to conduct an analysis of advertising-induced quantity and quality effects, household purchase data combined with generic advertising expenditures can be used. Household purchase data contain expenditures and quantities, along with other household characteristic variables. Unit values of the studied commodity are derived from the expenditure and quantity

variables and are used as proxies for prices. The derived unit value captures the quality of the purchased commodity, which is aggregated from many individual products under the commodity's category. Quality varies with the change of the composition of purchases made by the household of these individual products. Any change in product prices, household income, or advertising may induce the household to change the total amount of purchases of the commodity, or the purchase composition (quality), or both.

In this article, the effects of generic advertising on household purchases of fish and meat products are examined. Unlike previous advertising studies, our analysis investigates the impact of advertising on both quantity and quality of household purchases. The analysis is based on household purchase data from Norway. A censored demand system approach using fish and meat advertising expenditures as shifters of demand is used for the analysis. Unit values are used as proxies for the missing fish and meat prices in the data, and are treated as endogenous as functions of household characteristic, region, and advertising variables. This enables us to investigate not only the effect of advertising on demand, but also the effect of advertising on quality of purchases, as captured by the unit values.

The next section lays out the quality issue in household demand for meat and fish around the derivation of the effect of advertising on both quantity and quality. This is followed by a presentation of the econometric model that incorporates advertising and handles selectivity bias and quality. Finally, the

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results of the empirical model using Norwegian household data are presented and analyzed.

## 2. Theoretical issues of cross-section demand with unit values and advertising

Following Cox and Wohlgenant (1986), we assume that each of the commodities, fish and meat, form a separable branch of preferences and a Hicksian separable price structure. Thus, for each aggregate commodity  $j$ , which consists of different products, there exists a demand function that incorporates the effect of advertising as below:

$$Q_j = Q_j(V_j, V_s, a_j, a_s), \quad (1)$$

where  $Q_j$  is the quantity of the aggregate commodity  $j$  (e.g., fish) defined as the sum of quantities of products within commodity  $j$  each sold at a single price (e.g., salmon, cod, etc.).  $V_j$  is the unit value of commodity  $j$ ,  $V_s$  is a vector of the unit values of substitute commodities;  $a_j$  is advertising expenditures on commodity  $j$ , and  $a_s$  is a vector of advertising expenditures on substitute commodities.

The unit value of commodity  $j$  is derived by dividing the expenditures by the aggregated quantity:

$$V_j = \frac{E_j}{Q_j}, \quad (2)$$

where  $E_j$  is expenditure on commodity  $j$ , which is the sum of expenditures on all the products within commodity  $j$ . Both  $E_j$  and  $Q_j$  depend on the composition of households' purchases of the individual products constituting commodity  $j$ . As discussed by Deaton (1987, 1988, 1990), Nelson (1991), and Dong et al. (1998), the unit value  $V_j$  captures the quality of commodity  $j$  and is endogenous and determined by households' purchasing choice. We further assume that advertising affects unit value (quality) via influencing the composition of the quantities of the products chosen by the household for commodity  $j$ . Therefore, Eq. (1) can be modified as

$$Q_j = Q_j[V_j(a_j, a_s), V_s(a_j, a_s), a_j, a_s] \quad (3)$$

indicating that  $V_j$  and  $V_s$  also depend on  $a_j$  and  $a_s$ . Taking derivative of (3) with respect to  $a_j$ , we have,

$$\begin{aligned} dQ_j/da_j &= (\partial Q_j/\partial V_j)(\partial V_j/\partial a_j) \\ &\quad + (\partial Q_j/\partial V_s)(\partial V_s/\partial a_j) + \partial Q_j/\partial a_j, \end{aligned} \quad (4)$$

where  $\partial Q_j/\partial V_j$  and  $\partial Q_j/\partial V_s$  are the quantity effects of unit values,  $\partial Q_j/\partial a_j$  is the partial (direct) quantity effect of advertising, and  $\partial V_j/\partial a_j$  and  $\partial V_s/\partial a_j$  are the quality effects of advertising. In general,  $\partial Q_j/\partial V_j$  is negative, and  $\partial V_j/\partial a_j$  and  $\partial Q_j/\partial a_j$  are positive; while  $\partial Q_j/\partial V_s$  and  $\partial V_s/\partial a_j$  can be any sign. As a consequence, the sign of Eq. (4) is ambiguous, i.e., the total (net) effect of advertising on quantity is the mix of quantity effect of unit value, quality effect of advertising and the partial

quantity effect of advertising. The elasticity form of Eq. (4) can be expressed as follows:

$$\begin{aligned} \frac{d \ln Q_j}{d \ln a_j} &= \left( \frac{\partial \ln Q_j}{\partial \ln V_j} \right) \left( \frac{\partial \ln V_j}{\partial \ln a_j} \right) \\ &\quad + \left( \frac{\partial \ln Q_j}{\partial \ln V_s} \right) \left( \frac{\partial \ln V_s}{\partial \ln a_j} \right) + \left( \frac{\partial \ln Q_j}{\partial \ln a_j} \right). \end{aligned} \quad (5)$$

We can further simplify (5) as

$$\pi_j^T = \sum_{i=1}^M \psi_{ji}^Q \pi_{ji}^V + \pi_j^Q, \quad (6)$$

where  $\pi_j^T = \frac{d \ln Q_j}{d \ln a_j}$ , the total (net) quantity elasticity of advertising;  $\psi_{ji}^Q = \frac{\partial \ln Q_j}{\partial \ln V_i}$ , the  $j$ th quantity elasticity of unit value  $i$  (own unit value elasticity if  $j = i$  or cross-unit value elasticity if  $j \neq i$ );  $\pi_{ji}^V = \frac{\partial \ln V_j}{\partial \ln a_i}$ , the quality elasticity of advertising; and  $\pi_j^Q = \frac{\partial \ln Q_j}{\partial \ln a_j}$ , is the partial (direct) quantity elasticity of advertising.  $M$  in (6) represents the total number of commodities in the demand system. From Eq. (6), it is evident that quality effects can play an important role in the total demand response to advertising as measured in cross-section data.

## 3. Empirical model of censored demand system with unit values and advertising

Facing the same quality issue discussed in the previous section, Deaton (1987, 1990) proposed an econometric model based on the two set equations of demand and unit value, in which the quality elasticity and demand elasticity were estimated using household cluster data. In this study, we adopt a new model to correct for selectivity bias that is ignored by Deaton's approach. The model, following Deaton and Muellbauer (1980) and Pollack and Wales (1992), is an AIDS model based on the latent shares for  $M + 1$  commodities as follows:

$$S^* = A + \gamma \ln V + \xi \ln Y + \varepsilon, \quad (7)$$

where  $S^*$  is an  $M + 1$  column vector of latent expenditure shares on aggregate commodities,  $V$  is an  $M + 1$  column vector of commodity unit values, and  $Y = \frac{Y^*}{P^*}$ , the deflated total expenditures, with  $Y^*$  as total expenditures and  $P^*$  as a translog price index.  $\varepsilon$  is an  $M + 1$  column vector of equation error terms. The incorporating of household demographic variables and advertising is done through translating the intercept in (7).<sup>1</sup> That is, the intercept is defined as:  $A = \alpha + \beta X + \theta ADV_I$ , where  $X$  is an  $N$  column vector of demographic characteristics and  $ADV_I$  is the advertising variable that will be discussed later in the empirical application. Equation parameters are  $\xi [(M + 1) \times 1]$ ,  $\alpha [(M + 1) \times 1]$ ,

<sup>1</sup> In general, the incorporating of household demographic variables into the demand systems through the translating of the intercept affects total expenditures as well. See Pollak and Wales (1981), Blundell et al. (1993), and Gould et al. (1991). We follow Abdulai (2002) for simplicity.

$\beta [(M+1) \times N]$ , and  $\theta [(M+1) \times 1]$ ; and  $\gamma$  is an  $[(M+1) \times (M+1)]$  symmetric matrix.

Given the complexity of the problem, instead of using the nonlinear AIDS specification we use the linear approximate specification (LA/AIDS), where a linear approximation to  $\ln P^*$  is used as the expenditure deflator. Following Buse and Chan (2000), the invariant Tornqvist price index is used as a total expenditure deflator.

As was noted in the previous section, the price vectors  $P_j$  are not observed. Instead, the observed unit values ( $V_j$ ) are used as substitutes for the prices in (7). The unit value in (7) is treated as endogenous and defined as

$$\ln V = \delta Z + e, \quad (8)$$

where  $Z$  is an  $H$  column vector of variables such as regions and seasons to capture price variations or other variables, say advertising, that influence the household's choice of commodity's quality.  $\delta$  is an  $(M+1) \times H$  vector of parameters, and  $e$  is an  $M+1$  column vector of error terms.

Given the budget constraint, we know the latent shares must sum to one (adding up). This can be attained through parameter restrictions. Theoretical constraints such as homogeneity and symmetry can also be imposed on (7). The adding-up restriction implies that the joint density function of  $\varepsilon$  is singular. Consequently, one of the  $M+1$  latent share equations must be dropped during estimation. In dropping any equation from the estimation, we assume that the remaining  $M$  share equations' error terms,  $\varepsilon$  in (7), are distributed multivariate normal with a joint probability density function.

The mapping of the vector of latent shares,  $S^*$ , to observed shares,  $S$ , must take into account that the elements of  $S$  lie between 0 and 1, and sum to unity for each observation. The following mapping rule, from Wales and Woodland (1983), imposes these two characteristics:

$$S_i = \begin{cases} \frac{S_i^*}{\sum_{j \in \Delta} S_j^*}, & \text{if } S_i^* > 0, \\ 0, & \text{if } S_i^* \leq 0, \end{cases} \quad (i = 1, 2, \dots, M+1), \quad (9)$$

where  $\Delta$  is a set of all positive shares' subscripts. As Wales and Woodland (1983) point out, the way (9) maps  $S^*$  to  $S$  both is simple and has the property that the resulting density function is independent of whatever set of  $S^*$ s is used in its derivation. If any latent share happens to be negative, (9) will force the associated observed share to be zero and revalue all the positive shares.

### 3.1. Model estimation

Based on Dong et al. (2004),<sup>2</sup> assuming that at least one commodity is purchased, we can partition any observed purchase patterns into three general purchase regimes: (i) at least

one commodity is purchased, but the total number of purchased commodities is less than  $M$ , (ii)  $M$  commodities are purchased, or (iii) all  $M+1$  commodities are purchased. For each regime we can develop regime-specific likelihood functions that can be used to obtain system parameter estimates. Since a particular household is associated with only one purchase regime, the likelihood function appropriate for its purchase pattern determines the contribution this household makes to the overall sample likelihood function value.

#### Regime I Likelihood Function: Some Commodities Not Purchased

For households where  $k$  commodities are purchased and  $M > k \geq 1$ , we can rearrange the ordering of the  $M+1$  commodities so that the first  $k$  are purchased. We drop the last share equation. In this case Eq. (10) can be written as  $S^* = U^\omega + \omega$  with

$$U^\omega = A + \gamma_1 \ln V_1 + \gamma_0(\delta_0 Z) + \eta \ln Y, \quad (10)$$

where  $\gamma_1[k \times 1]$  is associated with the positive purchases,  $\gamma_0[(M+1-k) \times 1]$  is associated with the zero purchases,  $V_1$  is a vector of the observed unit values, and  $(\delta_0 Z)$  is a vector of the predicted unit values for the non-purchased commodities, and  $\omega = \varepsilon + \gamma_0 e_0$  represents the new error terms, where  $e_0$  is the error term of unobserved unit values. We assume  $\varepsilon \sim MN(0, \Sigma_{\varepsilon\varepsilon})$ , where  $\Sigma_{\varepsilon\varepsilon}$  is an  $[M \times M]$  error covariance matrix and is defined as

$$\Sigma_{\varepsilon\varepsilon} = \begin{bmatrix} \Sigma_{\varepsilon_1\varepsilon_1} & \Sigma_{\varepsilon_1\varepsilon_0} \\ \Sigma'_{\varepsilon_1\varepsilon_0} & \Sigma_{\varepsilon_0\varepsilon_0} \end{bmatrix}, \quad (11)$$

where  $\Sigma_{\varepsilon_1\varepsilon_1}$  is a  $k \times k$  error term covariance submatrix associated with the purchased commodities,  $\Sigma_{\varepsilon_0\varepsilon_0}$  is a  $(M-k) \times (M-k)$  covariance submatrix associated with the nonpurchased commodities, and  $\Sigma_{\varepsilon_1\varepsilon_0}$  is a  $(M-k) \times k$  submatrix of covariance across purchased and nonpurchased commodities. Considering the unit value equations, we further assume that the two sets of errors in (7) and (8) are jointly distributed normal with zero mean vector and variance covariance matrix as

$$\Sigma_\varepsilon = \begin{bmatrix} \Sigma_{\varepsilon\varepsilon} & \Sigma_{\varepsilon e} \\ \Sigma'_{\varepsilon e} & \Sigma_{ee} \end{bmatrix}, \quad (12)$$

where  $\Sigma_{\varepsilon e}$  is the covariance across share and unit value equations, which is defined as

$$\Sigma_{\varepsilon e} = \begin{bmatrix} \Sigma_{\varepsilon_1 e_1} & \Sigma_{\varepsilon_0 e_1} \\ \Sigma_{\varepsilon_1 e_0} & \Sigma_{\varepsilon_0 e_0} \end{bmatrix}, \quad (13)$$

and  $\Sigma_{ee}$  is the variance covariance matrix of the error terms of unit value equations and is defined as

$$\Sigma_{ee} = \begin{bmatrix} \Sigma_{e_1 e_1} & \Sigma_{e_1 e_0} \\ \Sigma'_{e_1 e_0} & \Sigma_{e_0 e_0} \end{bmatrix}. \quad (14)$$

<sup>2</sup> In their paper, unit values are treated as exogenous.

Then the joint distribution of  $\omega$  and  $e$  is  $MN(0, \Sigma)$ , where  $\Sigma$  is an  $[(2M + 1) \times (2M + 1)]$  error covariance matrix:

$$\Sigma = \begin{bmatrix} \Sigma_{\omega\omega} & \Sigma_{\omega e} \\ \Sigma'_{\omega e} & \Sigma_{ee} \end{bmatrix}, \quad (15)$$

where  $\Sigma_{\omega e} = (\Sigma_{\omega e_1}, \Sigma_{\omega e_0})$  with  $\Sigma_{\omega e_1} = \Sigma_{\varepsilon e_1} + \delta_0 \Sigma_{e_0 e_1}$ ,  $\Sigma_{\omega e_0} = \Sigma_{\varepsilon e_0} + \delta_0 \Sigma_{e_0 e_0}$ ,  $\Sigma_{\varepsilon e_1} = (\Sigma_{\varepsilon_1 e_1}, \Sigma_{\varepsilon_0 e_1})$ ,  $\Sigma_{\varepsilon e_0} = (\Sigma_{\varepsilon_1 e_0}, \Sigma_{\varepsilon_0 e_0})$ , and

$$\begin{aligned} \Sigma_{\omega\omega} &= \Sigma_{\varepsilon\varepsilon} + \delta_0 \Sigma_{e_0\varepsilon} + \Sigma_{\varepsilon e_0} \delta'_0 + \delta_0 \Sigma_{e_0 e_0} \delta'_0 \\ &= \begin{bmatrix} \Sigma_{11} & \Sigma_{10} \\ \Sigma'_{10} & \Sigma_{00} \end{bmatrix}, \end{aligned} \quad (16)$$

where  $\Sigma_{11}$  is a  $k \times k$  error term covariance submatrix associated with the purchased commodities,  $\Sigma_{00}$  is a  $(M - k) \times (M - k)$  submatrix associated with the nonpurchased commodities, and  $\Sigma_{10}$  is a  $(M - k) \times k$  submatrix of covariance across purchased and nonpurchased commodities.

Given (11)–(16), the likelihood of a household's being in a purchase regime where the first  $k$  commodities are positive and the remaining are zero can be represented via the following:<sup>3</sup>

$$\begin{aligned} L(S_1, S_2, \dots, S_k > 0; V_1, V_2, \dots, V_k > 0; \\ S_{k+1} = S_{k+2} \dots = S_{M+1} = 0 | \Theta) \\ = \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \int_{S_1}^{+\infty} \int_{1-\frac{S_1^*}{S_1}}^0 \int_{1-\frac{S_1^*}{S_1}-\frac{S_2^*}{S_2}}^0 \dots \int_{1-\frac{S_1^*}{S_1}-\frac{S_{k+1}^*}{S_{k+1}}-\dots-\frac{S_{M-1}^*}{S_{M-1}}}^0 \\ \times \phi(S_1^*, S_2^*, \dots, S_M^*; U^\omega, \Sigma_{\omega\omega}) dS_M^* \dots dS_{k+1}^* dS_1^*, \end{aligned} \quad (17)$$

where  $\Theta$  contains of all the parameters to be estimated,  $\phi(\cdot)$  is the  $M$ -dimension  $PDF$  of latent shares, and  $\varphi(\cdot)$  is the  $k$ -dimension  $PDF$  of the errors of the observed unit values given the  $k$  positive shares with the mean vector of

$$e_{1|S} = \ln V_1 - \delta_1 Z - \Sigma'_{\omega e_1} \Sigma_{\omega\omega}^{-1} S^1, \quad (18)$$

and the error covariance matrix,

$$\Sigma_{e_{1|S}} = \Sigma_{e_1 e_1} - \Sigma'_{\omega e_1} \Sigma_{\omega\omega}^{-1} \Sigma_{\omega e_1}, \quad (19)$$

where  $\delta_1$  is the vector of parameters associated with the  $[k \times 1]$  observed unit values defined in (18), and  $S^1 = (S_1, S_2, \dots, S_k)'$ , the  $[k \times 1]$  vector of the positive shares.

Equation (17) is based on the mapping defined by (9). The integral in (17) is  $[M - k + 1]$ -fold, i.e., the number of non-purchased commodities plus one. As noted above, if the demand system encompasses a large number of commodities and there are a large number of nonpurchased commodities for a particular household, the conventional method for numerically evaluating (17) is impractical. However, (17) can be evaluated using a number of alternative simulation procedures. For

the present analysis we use the smooth recursive conditioning simulator (GHK) suggested by Geweke (1991), Hajivassiliou et al. (1996), and Keane (1994). The GHK procedure requires that (17) be a rectangular standard multivariate normal probability. The current representation of (17) does not satisfy this requirement. However, (17) can be stated in a form that can be simulated using the GHK algorithm, as follows:

$$\begin{aligned} L(S_1, S_2, \dots, S_k > 0; S_{k+1} = S_{k+2} \dots = S_{M+1} = 0) \\ = B \cdot \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \cdot \Phi_{M-k+1}(b; R_C), \end{aligned} \quad (20)$$

where  $\Phi_{M-k+1}(b; R_C)$  is a  $[M - k + 1]$  dimensional multivariate standard normal  $cdf$  evaluated at vector  $b$  with correlation coefficient matrix  $R_C$ . Note that  $\Phi_{M-k+1}(b; R_C)$  is an  $[M - k + 1]$ -fold probability integral. The detailed transformation of (17) to (20), with the definitions of the matrices  $b$ ,  $R_C$ , and  $B$ , is presented in Appendix A.

#### Regime II Likelihood Function: One Commodity Not Purchased

In Regime II, the number of commodities actually purchased,  $k$ , equals  $M$ . Under this special case, Eq. (17) can be simplified as

$$\begin{aligned} L(S_1, S_2, \dots, S_M > 0; S_{M+1} = 0) \\ = B_1 \cdot \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \int_{S_1}^{+\infty} \phi(S_1^*; U^*, \Omega_{11}) dS_1^*. \end{aligned} \quad (21)$$

Appendix A shows the derivation of (21) and the definition of  $B_1$ ,  $U^*$ , and  $\Omega_{11}$ . Equation (21) implies that under Regime II, the likelihood function requires only the integration of a univariate  $PDF$ .

#### Regime III Likelihood Function: All Commodities Purchased

For households where all commodities are purchased ( $k = M + 1$ ), the likelihood function is just the  $[(2M + 1) \times 1]$  multivariate  $PDF$  of joint error terms,  $\varepsilon$  and  $e$ , which are defined in (7) and (8), and distributed as  $MN(0, \Sigma_\varepsilon)$ . That is

$$L(S_1, S_2, \dots, S_{M+1} > 0) = \phi(\varepsilon, e). \quad (22)$$

Consistent and efficient parameter estimates can be obtained by maximizing the sum of log likelihood function over all households, where each household has been associated with one of the above regime-specific likelihood functions.

### 3.2. Elasticity evaluation

#### 3.2.1. Unit value

Elasticities are evaluated based on the expected values. To obtain the expected values of observed expenditure shares, we need to calculate the expected unit values. Similar to the single equation case studied by Dong et al. (1998) the expected unit values can be obtained as

$$E(\ln V) = \delta Z. \quad (23)$$

<sup>3</sup> The Jacobian between latent shares ( $S^*$ ) and observed shares ( $S$ ) is ignored because it is independent of model parameters. For details, see Wales and Woodland (1983).

Thus, the elasticities of unit value with respect to the exogenous variable  $Z$  evaluated at its sample mean  $\bar{Z}$  can be calculated as

$$\eta_Z^V = \delta \bar{Z}. \quad (24)$$

Expected values of observed expenditure shares can be obtained by summing the product of each regime's probability and the expected conditional share values over all possible regimes. Let  $R_s$  represent a particular purchase regime:

$$R_s = (S_1 = S_2 = \dots = S_s = 0; S_{s+1} > 0, \dots, S_{M+1} > 0). \quad (25)$$

The expected value of the  $j$ th observed expenditure share is:<sup>4</sup>

$$E(S_j) = \sum_{s=1}^M \alpha_{R_s} E(S_j | R_s), \quad (26)$$

where  $\alpha_{R_s}$  is the probability that regime  $R_s$  occurs. The expected share value conditional on purchase regime  $R_s$  can be represented as

$$E(S_j | R_s) = \begin{cases} E \left[ \frac{(S_j^* | R_s)}{\sum_{i=s+1}^{M+1} (S_i^* | R_s)} \right], & \text{if } j > s, \\ 0, & \text{if } j \leq s. \end{cases} \quad (27)$$

From (26) the impact of changes in unit values, advertising, demographic characteristics, or total expenditures on food demand can be obtained, but one needs to evaluate  $M$ -dimension integrals. Given that there are  $2^{M+1}-1$  purchase regimes, one may need to evaluate these integrals a large number of times for a reasonably sized demand system.

Phaneuf et al. (2000) developed a simulation procedure to evaluate the elasticities for a censored demand system applied to recreation choices. We adapt their procedure to our application. Assume we have  $R$  replicates of the  $[M+1]$  error term vectors  $\varepsilon$  in (7) and  $e$  in (8). The  $r$ th simulated latent share vector,  $S_r^*$ , evaluated at the sample means of our exogenous variables (indicated by a bar over a variable) is

$$S_r^* = \alpha + \gamma \ln \bar{V} + \beta \ln \frac{\bar{Y}}{P^*} + \varepsilon_r, \quad (28)$$

where  $\varepsilon_r$  is the  $r$ th replicate of  $\varepsilon$ . The  $r$ th replicate of the  $i$ th observed share then is

$$S_{ir} = \begin{cases} \frac{S_{ir}^*}{\sum_{j \in \Delta} S_{jr}^*}, & \text{if } S_{ir}^* > 0, \\ 0, & \text{if } S_{ir}^* \leq 0. \end{cases} \quad (29)$$

<sup>4</sup> This is the regime where the first  $s$  shares are zero. Given  $s$  zero-valued shares, other possible purchase patterns can be transformed to this pattern by rearranging the share ordering. Under this definition, regime  $R_s$  is actually the sum of all the purchase patterns with  $s$  zero-valued shares.

The expected observed share vector for  $R$  replicates is then calculated as a simple average of these simulated values

$$E(S) = \frac{1}{R} \sum_{r=1}^R S_r. \quad (30)$$

If there is a small change in unit value  $j$ ,  $\Delta V_j$ , then the elasticity vector with respect to this unit value change is

$$\psi_j^Q = -\Lambda_j + \frac{\Delta E(S)}{\Delta E(V_j)} \cdot \frac{E(V_j) + \Delta E(V_j)/2}{E(S) + \Delta E(S)/2}, \quad (31)$$

where  $\Lambda_j$  is a vector of 0's with the  $j$ th element equal to 1, and  $\Delta E(S)$  is the change in the simulated  $E(S)$  given the change of expected unit value,  $\Delta E(V_j)$ , and  $E(V_j)$  is derived from (23).

### 3.2.2. Price

As mentioned above, the cross-sectional price is not directly observed. However, the price elasticity can be derived from the unit value and quality elasticities. Since unit value captures price and quality, we can assume:

$$V_j = P_j W_j, \quad (32)$$

where  $P_j$  is the price index of commodity  $j$  and  $W_j$  is a quality indicator of commodity  $j$ . Following Deaton (1987, p. 11), we can derive the following elasticity relationships:

$$\varepsilon_{jk}^V = \delta_{jk} + \frac{\eta_j^W \varepsilon_{jk}^Q}{\eta_j^Q}, \quad (33)$$

where  $\varepsilon_{jk}^V = \frac{\partial \ln V_j}{\partial \ln P_k}$  and  $\varepsilon_{jk}^Q = \frac{\partial \ln Q_j}{\partial \ln P_k}$  are the unit value and quantity elasticities, respectively, of commodity  $j$  with respect to the price index of commodity  $k$ .  $\eta_j^W = \frac{\partial \ln W_j}{\partial \ln E_j}$  and  $\eta_j^Q = \frac{\partial \ln Q_j}{\partial \ln E_j}$  are the quality and quantity elasticities, respectively, of commodity  $j$  with respect to total expenditure, and  $\delta_{jk}$  is the Kronecker delta. The second term in the right-hand side of Eq. (33) is  $\varepsilon_{jk}^W = \frac{\partial \ln W_j}{\partial \ln P_k}$ , the quality elasticity of commodity  $j$  with respect to the price of commodity  $k$ . In practice, total expenditure is assumed to have no effect on price. Therefore,  $\eta_j^W$  is the same as  $\eta_j^V = \frac{\partial \ln V_j}{\partial \ln E_j}$ , the unit value elasticity of commodity  $j$  with respect to total expenditure. Through the relationship  $\varepsilon_{jk}^Q = \psi_{jk}^Q \cdot \varepsilon_{jk}^V$  (where  $\psi_{jk}^Q = \frac{\partial \ln Q_j}{\partial \ln V_k}$  is the quantity elasticity of commodity  $j$  with respect to the unit value of commodity  $k$ ) we have:

$$\varepsilon_{jk}^Q = \begin{cases} \psi_{jj}^Q / [1 - \psi_{jj}^Q (\eta_j^V / \eta_j^Q)], & \text{if } k = j \\ \psi_{jk}^Q [1 + \varepsilon_{kk}^Q (\eta_j^V / \eta_j^Q)], & \text{if } k \neq j. \end{cases} \quad (34)$$

If we use the unit value ( $V_j$ ) as a proxy for the unobserved price ( $P_j$ ) in the demand system, and simultaneously make the unit value endogenous as a function of total expenditure, the unobserved price elasticity ( $\varepsilon_{jk}^Q$ ) can be recovered using Eq. (34).

### 3.2.3. Advertising

The advertising elasticities have been analyzed above and the total quantity elasticity of advertising can be calculated by using Eq. (6) if the data are not censored. In this study, Eq. (6) cannot be used directly due to the data censoring. In practice, similar like the price, we simulate the advertising elasticities using Eqs. (23) and (26).

## 4. Norwegian household fish and meat demand and the effects of advertising

Per capita consumption of fish and meat in Norway has increased in the past two decades, but it is still lower than in other Nordic countries. Household preference for fish and meats in Norway has also shifted to higher quality products. For example, pork is substantially leaner than it used to be. The leaner the pork is, the higher the quality. This shift may be due to increases in household income, nutrition and health concerns, but may also due to advertising.

Two studies of Norwegian demand for fish and meat can be found in the literature (Rickertsen, 1996; Rickertsen et al., 2003). Both used annual commercial disappearance data. However, the effect of advertising on Norwegian domestic demand for fish and meat has not yet been analyzed. Hence, an additional empirical contribution of our study is an analysis of the direct and spillover effects of generic fish and meat advertising in Norway.

### 4.1. Data

The data used in this article are drawn from a panel survey of more than 1,500 Norwegian households provided by GfK Norge, a marketing research company. These households report the expenditure and quantity of each fish and meat item purchased at every shopping trip made during the week. Each household also provides demographic characteristics, such as household size, age of the head of the household, location, and income. The data used for estimation contain weekly household purchase information for fish and meat products for January 1999 through December 2000. Since the generic fish and meat advertising data are recorded as monthly expenditures, the purchase data are reformulated on a monthly basis and merged with the advertising data. The advertising data vary over time but not across households.

The final demand system consists of four aggregated commodities: fish, beef, pork, and other meat. The fish commodity is aggregated from cod, salmon, farmed fish, prawns, canned mackerel, and canned, or bucket herring. Purchase statistics for these commodities are provided in Table 1. Of these four commodities, beef is purchased most frequently (87%), followed by fish (69%), pork (54%), and other meat (15%). For the expenditure share among the four commodities, beef accounts for 54% and other meat accounts for only 4.8%. The three meat commodities in total account for about 78% of the combined fish

Table 1

Expenditure share, unit value, and purchase frequency

Commodity	Mean share	Mean unit value over purchase occasions (standard error)	Purchase frequency
Fish	0.2240	57.251 (32.031)	0.6923
Beef	0.5392	63.947 (22.689)	0.8687
Pork	0.1889	80.788 (41.145)	0.5435
Other meat	0.0479	72.537 (51.424)	0.1532

Table 2

Explanatory variables used in share and unit value equations

Name	Description	Unit	Means
<i>Household characteristics</i>			
TOTEXP	Monthly total fish and meat expenditures	NOK	266.3
HSIZE	Household size	number	1.840
AGE_HEAD	Age of household head	number	49.62
NUM.KID	Proportion of persons under 16	%	0.3828
<i>Regions</i>			
METRO	Dummy vs. rural	0/1	0.7550
NORTH	Region dummy	0/1	0.1045
CENTRAL	Region dummy	0/1	0.1651
WEST	Region dummy	0/1	0.1945
OSLO	Region dummy	0/1	0.1299
<i>Advertising</i>			
ADV_FISH	Sum of weighted lag fish advertising	NOK million	6.067
ADV_MEAT	Sum of weighted lag meat advertising	NOK million	24.037

and meat expenditures, while the fish commodity accounts for 22%. The observed unit values vary from 57.3 NOK/kg paid for fish to 80.8 NOK/kg paid for pork. The unit value of other meat has the largest variation, as evidenced by its standard deviation, while beef has the smallest.

Table 2 provides an overview of the explanatory variables used in the share and unit value equations. As defined in the AIDS specification, total expenditure and unit values (in place of the unobserved prices) are included in the share equations. Household demographic variables and advertising are incorporated through the intercept, as suggested by Pollack and Wales (1992). The same set of household demographic variables is adopted in the unit value equations.

Advertising used in this analysis reflects total monthly generic advertising expenditures for meat and fish. In our data, advertising expenditures for the three meat commodities cannot be separated. Thus, we use only one advertising measure, the combined expenditure, for the three meat commodities. To capture the carry-over effect of advertising, advertising expenditures are lagged nine months and a polynomial distributed lag model is adopted as follows (Clarke, 1976):

$$ADV\_I_t = \sum_{i=0}^L \lambda_i D_{t-i}, I = \text{Fish or Meat}, \quad (35)$$

where  $D_{t-i}$  is the  $i$ th lag of advertising at time  $t$ ,  $L$  is the total lag length, which is nine in this case (following the suggestion of Clarke, 1976), and  $\lambda_i = \theta_0 + \theta_1 i + \theta_2 i^2$  ( $i = 0, 1, \dots, L$ ) are the quadratic weights of lagged advertising. Two restrictions are imposed on  $\lambda_i$ : (i) current advertising has the maximum weight, which is defined as one ( $\lambda_0 = 1$  as the maximum); (ii) the weight of the tenth lag is zero ( $\lambda_{10} = 0$ ), that is, the effect of advertising ends at the tenth month (i.e., has a nine-month-lag's effect). After imposing restrictions (i) and (ii), we have  $\lambda_i = 1 - \frac{1}{(L+1)^2} i^2$ .  $ADV I_t$ , the sums of weighted advertising over the current and all lagged periods for both fish and meat, are used as explanatory variables in the demand and unit value equations. The coefficients of  $ADV I_t$  represent the long-run effect of advertising.

#### 4.2. Estimated coefficients

Parameter estimates were obtained by maximum likelihood estimates of the model using the GAUSS software system. The standard errors of the estimated coefficients are obtained from the inverse of the negative numerically evaluated Hessian matrix of the likelihood function. The estimated coefficients are presented in Appendices B and C.

The other meat commodity is dropped from the estimation because of the adding-up condition. The estimated coefficients associated with other meat are retrieved using this condition imposed on the share system. The Cholesky decomposition of the variance covariance matrix of share equations is estimated since it guarantees a positive definite variance covariance matrix during the entire estimation process. The unit value equations, unlike the share equations, do not require the adding up condition. The four unit value equations are estimated simultaneously with the three share equations (one is dropped). The values of the estimated coefficients are of little interest because of the overlapping effects of the explanatory variables, and therefore we will not discuss them. From these coefficients, however, the elasticities of each explanatory variable are computed.

#### 4.3. Estimated elasticities

The computed elasticities of explanatory variables including advertising evaluated at the sample mean are given in Tables 3, 4, and 5. Table 3 provides the demand elasticity of unit value ( $\psi_{jk}^Q$ ) and the unobserved price elasticity ( $\varepsilon_{jk}^Q$ ) calculated by using Eqs. (31) and (34), respectively. The quality elasticity ( $\eta_j^V$ ), and expenditure elasticity ( $\eta_j^Q$ ) are also provided in Table 3. To better understand the effects of advertising, we separate the advertising elasticities from other variables and report them in Table 4. The total (net) quantity elasticities of advertising are calculated by using Eq. (6). Table 5 provides the elasticities of exogenous variables for unit value ( $V$ ), the demand when unit value is given ( $Q|V$ ) (i.e., the unit value is fixed and the household is not allowed to adjust its purchase composition for quality), and the demand when unit value is not given ( $Q$ ). The

Table 3  
Unit value and unobserved price elasticities

Elasticity		Commodity			
		Fish	Beef	Pork	Other meat
Unit value ( $\psi_{jk}^Q$ )	Fish	−0.7308*	−0.1561*	0.0825	0.1407
	Beef	−0.3843*	−0.8302*	0.3460*	−0.2188
	Pork	0.0228	0.0452*	−1.1074*	−0.0649
	Other meat	0.0924	−0.0589	−0.0097	−0.8570*
Price ( $\varepsilon_{jk}^Q$ )	Fish	−0.6774*	−0.1499*	0.0783	0.1316
	Beef	−0.3515*	−0.7914*	0.3255*	−0.2022
	Pork	0.0203	0.0425*	−1.0227*	−0.0600
	Other meat	0.0845	−0.0561	−0.0091	−0.7920*
Quality ( $\eta_j^V$ )		0.0700*	0.0594*	0.0957*	0.1024
Expenditure ( $\eta_j^Q$ )		0.6484*	1.0072	1.2801*	1.0700*

\*indicates statistically significant at 0.05 level or higher. The standard errors of the elasticities are derived from Delta Method (Rao, 1973).

Table 4  
Comparison of various advertising elasticity measures

Elasticity	Commodity	Fish advertising	Meat advertising
Total (net) quantity elasticities of advertising ( $\pi_j^T$ )	Fish	0.1286*	−0.1335
	Beef	−0.0523*	0.2149*
	Pork	−0.0128	−0.1724*
	Other meat	0.0534	−0.5002
Partial (direct) quantity elasticities of advertising ( $\pi_j^Q$ )	Fish	0.1976*	0.1549
	Beef	−0.0929*	0.0140*
	Pork	−0.0020	−0.0608
	Other meat	0.1111	−0.2277*
Quality elasticities of advertising ( $\pi_{ji}^V$ )	Fish	0.0050	0.0403
	Beef	−0.0440	0.0569
	Pork	−0.2100	0.2407*
	Other meat	−0.1219	−0.0497
Expenditure share	Fish	0.1336*	−0.0932
	Beef	−0.0963*	0.2718*
	Pork	−0.2228	0.0683*
	Other meat	−0.0685	−0.5499*
Total expenditure		−0.0674	0.1123

\*indicates statistically significant at 0.05 level or higher. The standard errors of the elasticities are derived from Delta Method (Rao, 1973).

results in the columns under ( $Q|V$ ) can be viewed as the partial (direct) quantity effects of exogenous variables, and the results under ( $V$ ) are quality effects. The total (net) effects of the two on purchases (demand) are given under the column heading ( $Q$ ). This total effect is a combination effect of an exogenous variable acting directly on the purchase and its indirect effect through the change in unit value (quality).

##### 4.3.1. Unit value and price

Table 3 provides the unit value elasticities and the recovered uncompensated price elasticities. As in Deaton (1987), the relatively small quality elasticities ( $\eta_j^V$ ) result in small

Table 5  
Estimated various elasticities of exogenous variables

	(Q   V)				(V)				(Q)			
	Fish	Beef	Pork	Other meat	Fish	Beef	Pork	Other meat	Fish	Beef	Pork	Other meat
<i>Total expenditure</i>												
TOTEXP	0.6484*	1.0072*	1.2801*	1.0700*	0.0700*	0.0594*	0.0957*	0.10248	0.6272*	1.0038*	1.1723*	0.9800*
<i>Demographic variables</i>												
HSIZE	0.2752*	−0.0405*	−0.1297*	−0.0472	−0.0780	−0.0295	−0.1728*	−0.0764	0.2545*	−0.0367*	−0.1178*	−0.0514
AGE.HEAD	0.7026*	−0.3523*	0.0415	0.4143*	0.0385	−0.0721*	−0.2605*	0.0572	0.7400*	−0.3857*	0.0696*	0.4606*
NUM.KID	−0.0005	0.0156	−0.0124	−0.0027	−0.2473	−0.2944*	0.0956	0.0758	−0.0003	0.0150	−0.0125	−0.0023
METRO	0.0092	−0.0104	−0.0149*	−0.0161*	0.0857	0.3663*	0.5084	−1.5374*	0.0036	−0.0012	−0.0138*	0.0114*
NORTH	−0.0500*	0.0010	0.0378*	0.0112	−0.0199	0.6381*	0.6631	1.7918*	−0.0474*	0.0007	0.0345*	0.0121
CENTRAL	−0.0347*	−0.0014	0.0343*	0.0018	0.0237	0.1099	1.8388*	1.3217	−0.0304*	−0.0030	0.0303*	0.0031
WEST	−0.0278*	−0.0054	0.0331*	0.0002	0.3757	0.3072	2.3290*	2.5806*	−0.0210*	−0.0122	0.0298*	0.0034
OSLO	0.0351*	−0.0328	−0.0033	0.0010	0.2482	0.2958	2.0487	0.1850	0.0343*	−0.0311	−0.0038	0.0006

\*indicates statistically significant at 0.05 level or higher. The standard errors of the elasticities are derived from Delta Method (Rao, 1973).

differences in price elasticities compared with unit value elasticities. All the own price elasticities are negative and statistically significant. The cross-price elasticities of fish versus beef and beef versus pork are also found to be statistically significant, and their signs indicate that fish and beef are gross complements, and that beef and pork are gross substitutes. The results also show that the price elasticity of pork demand is elastic, while those of the other products are inelastic. The elasticity values are quite comparable to those estimated by Rickertsen (1996).

#### 4.3.2. Advertising

The results in Table 4 show that a 1% increase in fish advertising significantly increases fish purchases directly by 0.2%. This is the partial effect, i.e., when the quality effect is not taken into account. The own quality effect of fish advertising on the unit value for fish is positive (0.005%), but insignificant. Hence, it appears that generic fish advertising has a larger quantity effect than a quality effect. Given the partial effect of fish advertising on fish purchases (0.2%) and the quality effect of fish advertising on the unit value of both own (fish: 0.005%), and cross (beef: −0.04%, pork: −0.21%, and other meat: −0.12%), the total quantity effect on fish purchases using Eq. (6) can be derived as 0.13%. Thus, ignoring quality effects would cause the total advertising elasticity for fish to be overstated by 54%. This overstatement has important implications for optimal advertising expenditure levels. The relatively small total (net) effect (0.13%) compared with the partial (direct) effect (0.2%) is due to the compound term in the right hand side of Eq. (6) summing to a negative amount. The negative amount is from the small changes in unit values (qualities), which, in turn, decrease the quantity of fish purchases through the cross and own unit value effects. It offsets the partial purchase effect induced by advertising.

Fish advertising is also found to have a significant and negative spillover effect on beef purchases. After accounting for the quality effects, the total effect of fish advertising on beef

purchases is still significant and negative. However, there are no significant effects of fish advertising on pork and other meat demand.

Meat advertising has a significant and positive partial quantity effect on beef, but an insignificant positive effect on beef quality (unit value). After accounting for the cross and own unit value effects, the total (net) effect of meat advertising on beef purchases is still significant, but more positive. Hence, meat advertising also has a stronger quantity effect on beef than a quality effect.

The effect of meat advertising on pork is a different story. Meat advertising is found to have a negative but insignificant partial quantity effect on pork. However, meat advertising has a positive and statistically significant effect on pork quality. Consequently, the total effect of meat advertising on pork purchases is significant and negative. Hence, meat advertising induces households to purchase less, but higher quality pork. Meat advertising does not have any significant total effects on fish or other meat demand.

In order to see the effects of generic advertising on fish and meat expenditures, we compute the advertising elasticities of expenditure share, which are also given in Table 4. These elasticities are the sum of the total quantity elasticities of advertising ( $\pi_j^T$ ) and the quality elasticities of advertising ( $\pi_j^V$ ). The two numbers in the “Total expenditure” column are the sums of the advertising elasticities of expenditure share multiplied by their corresponding shares, which give the effects of advertising on combined fish and meat expenditures.

The results show that generic fish advertising significantly increases fish expenditures, but at the same time decreases expenditures on meats. As a consequence, the effect of fish advertising on combined fish and meat expenditures is negative. In particular, a 1% increase in fish advertising will increase household fish expenditures by 0.13%, and decrease beef expenditures and combined fish and meat expenditures by 0.1% and 0.07%, respectively. Meat advertising significantly increases beef and pork expenditures, even though it

decreases other meat and fish expenditures. As a result, combined fish and meat expenditures are increased by generic meat advertising.

#### 4.3.3. Total expenditure, household characteristics, and region variables

The numbers in the columns under ( $Q|V$ ) in Table 5 are the elasticities of the direct (partial) quantity effects of exogenous variables on fish and meat. Total expenditure is found to be significant for all four commodities, with only the expenditure elasticity of fish being less than one. Since the system we estimated includes only fish and meat, the elasticities are conditional on the predetermined expenditure for this subsystem. The calculated unconditional expenditure elasticities are all less than one, indicating that none of the four commodities is a luxury good.<sup>5</sup>

The direct quantity effect of household size is found to be significant for all commodities except for other meat. Size of household is positively related with fish purchases (0.28), but negatively related with beef (−0.04) and pork (−0.13) purchases. The head of the household's age has a significant effect on the quantity purchased for fish, beef, and other meat, but not for pork; age is positively related with fish and other meat purchases, but negatively related with beef purchases. This variable is actually the most important one since it gives the largest elasticity for fish (0.70), beef (−0.35), and other meat (0.41) among all the household characteristic and region variables. The results also show that households in metropolitan areas purchase less pork and other meat relative to households in rural areas. Regional effects are found to be significant for fish and pork only. Relative to the East, households in the North, Central, and West regions purchase less fish, while households in Oslo purchase more fish. Pork demand is higher in the North, Central, and West compared to the East. No significant effect is found for the number of children in the household.

Total fish and meat expenditure has a positive effect on the quality (unit value) for all four commodities. The largest effect is found for the other meat category, followed by pork. These two commodities also have the highest average unit values and the largest unit value standard errors. The effect of household size on unit value is negative across all commodities, but only for pork is it statistically significant. This indicates that large households purchase less expensive pork products than small households. The effects of age on unit values of beef and pork are found to be significant and negative, indicating that older heads of households have a propensity to consume less expensive beef and pork products compared to younger household heads. Households with more children

tend to purchase cheaper beef products compared to households with fewer children. Households in metropolitan areas pay more for beef products and less for other meat items relative to those in rural areas. The regional effect on pork and other meat unit values is also notable. Households in the West pay higher unit values for these two commodities relative to households in the East (base). The results also show that households in the Central region pay relatively higher unit values for pork, and households in the North pay higher unit values for beef.

Looking at the total (net) effects on fish and meat purchases (columns under ( $Q$ ) in Table 5), we find that total expenditures have a pattern similar to the direct quantity effects. But in this case, the value of other meat is less than one (0.98). However, after transforming them to the unconditional elasticities, as was done for the direct quantity effects, they all become less than one. The effect of household size and the age of the head of the household are similar to the direct effects, but under this case, the effect of age on pork becomes significant. Being in a metropolitan area shows a significant and negative total effect on the demand for pork and a negative total effect on the demand for other meat unlike the case for direct effect. Fish and pork demand is significantly influenced by region, but beef and other meat demand is not, which is similar to the direct effect results.

## 5. Summary

In this article we estimated the demand for fish, beef, pork, and other meat commodities using Norwegian household data via a censored demand system approach. In the demand model, generic fish advertising and meat advertising were introduced as demand shifters, and at the same time we treated unit values as endogenous. The unit values capture the quality of the commodities. The main purpose of this article was to investigate quantity and quality effects of advertising on household demand.

Our results showed that generic fish advertising had a significant effect on increasing fish quantity, but that its effect on quality was insignificant. Fish advertising also decreased beef quantity, but had no significant impact on quality. Generic meat advertising also had a significant impact on increasing beef quantity, but its effect on quality was insignificant. However, generic meat advertising did not have a significant quantity effect on pork demand, but did have a significant positive quality impact on pork.

Generic fish advertising was found to significantly increase fish expenditures, but at the same time decrease meat expenditures, i.e., fish advertising has a negative spillover effect on meats. As a consequence, the effect of fish advertising on combined fish and meat expenditures was negative. However, combined fish and meat expenditures were increased by generic meat advertising.

<sup>5</sup> Since we assume weak separability, to get the unconditional expenditure elasticities we multiply the income elasticity of 0.75 estimated by Rickertsen (1994) for animal products by the estimated conditional expenditure elasticities in Table 3.

## Appendix A. Derivation of the estimable likelihood functions<sup>6</sup>

The likelihood function in (20) can be decomposed into two components, in a procedure similar to that shown by Pudney (1989, pp. 327–328). Our case is more complicated because of adding-up restrictions on both  $S^*$  and  $S$ . Below is a simplification of (20) in which we reduce the dimension of  $\phi(\cdot)$  from  $M$  to  $[M - k + 1]$ :

$$\begin{aligned} L(S_1, S_2, \dots, S_k > 0; S_{k+1} = S_{k+2} \dots = S_{M+1} = 0) \\ = B \cdot \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \int_{S_1}^{+\infty} \int_{1-\frac{S_1^*}{S_1}}^0 \int_{1-\frac{S_1^*}{S_1}-S_{k+1}^*}^0 \dots \int_{1-\frac{S_1^*}{S_1}-S_{k+1}^*-\dots-S_{M-1}^*}^0 \\ \times \phi(S_1^*, S_{M+1}^*, \dots, S_M^*; U^*, \Omega_{11}) dS_M^* \dots dS_{k+1}^* dS_1^* \quad (A.1) \end{aligned}$$

where

$$U^* = \begin{pmatrix} U_1^* \\ U_{k+1}^* \\ \vdots \\ U_M^* \end{pmatrix} = \Omega_{11} \Omega_{10}^{-1} \begin{pmatrix} U_1 \\ U_{k+1} \\ \vdots \\ U_M \end{pmatrix},$$

an  $[(M - k + 1) \times 1]$  vector, and

$$\begin{aligned} B = (2\pi)^{\frac{1-k}{2}} \cdot |\Sigma|^{-\frac{1}{2}} \cdot |\Omega_{11}|^{\frac{1}{2}} \\ \cdot e^{-\frac{1}{2} \left\{ \begin{pmatrix} U_1 \\ U_{k+1} \\ \vdots \\ U_M \end{pmatrix}' \Omega_{00}^{-1} \begin{pmatrix} U_1 \\ U_{k+1} \\ \vdots \\ U_M \end{pmatrix} - \begin{pmatrix} U_1^* \\ U_{k+1}^* \\ \vdots \\ U_M^* \end{pmatrix}' \Omega_{11}^{-1} \begin{pmatrix} U_1^* \\ U_{k+1}^* \\ \vdots \\ U_M^* \end{pmatrix} \right\}} \end{aligned}$$

Vector

$$\begin{pmatrix} U_1 \\ U_{k+1} \\ \vdots \\ U_M \end{pmatrix}$$

is from  $U^\omega$  defined in (13).

The above  $\Omega_{ij}$ 's are  $[(M - k + 1) \times (M - k + 1)]$  matrixes, and defined as

$$\begin{aligned} \Omega_{11} = \begin{bmatrix} I' \sigma_{11} I & I' \sigma_{10} \\ \sigma'_{10} I & \sigma_{00} \end{bmatrix}, \quad \Omega_{00} = \begin{bmatrix} J' \sigma_{11} J & J' \sigma_{10} \\ \sigma'_{10} J & \sigma_{00} \end{bmatrix}, \quad \text{and} \\ \Omega_{10} = \begin{bmatrix} I' \sigma_{11} J & I' \sigma_{10} \\ \sigma'_{10} J & \sigma_{00} \end{bmatrix}, \end{aligned}$$

where  $I$  is a  $[k \times 1]$  vector of ones, and  $J$  is a  $[k \times 1]$  vector with the elements:

$$\left( 1, \frac{U_2}{\left(\frac{s_2}{s_1}\right) U_1}, \frac{U_3}{\left(\frac{s_3}{s_1}\right) U_1}, \frac{U_4}{\left(\frac{s_4}{s_1}\right) U_1}, \dots, \frac{U_k}{\left(\frac{s_k}{s_1}\right) U_1} \right)'.$$

The  $\sigma_{ij}$ 's are defined via the following  $[M \times M]$  matrix:

$$\begin{bmatrix} \sigma_{11} & \sigma_{10} \\ \sigma'_{10} & \sigma_{00} \end{bmatrix} = \begin{bmatrix} A \Sigma_{11}^{-1} A' & A \Sigma_{10}^{-1} \\ \Sigma_{10}^{-1'} A' & \Sigma_{00}^{-1} \end{bmatrix}^{-1}.$$

The  $[k \times k]$  matrix  $A$  is a diagonal matrix with elements:

$$\left( 1, \frac{S_2}{S_1}, \frac{S_3}{S_1}, \dots, \frac{S_k}{S_1} \right).$$

Finally, the  $\Sigma_{ij}^{-1}$  matrices are obtained from the full error variance matrix,  $\Sigma_{\omega\omega}$ , in (19).

From the results shown in Tallis (1965), the likelihood function represented by (A.1) can be further transformed to:

$$\begin{aligned} L(S_1, S_2, \dots, S_k > 0; S_{k+1} = S_{k+2} \dots = S_{M+1} = 0) \\ = B \cdot \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \cdot \Phi_{M-k+1}(b; R_C), \quad (A.2) \end{aligned}$$

where  $\Phi_{M-k+1}(b; R_C)$  is a  $[M - k + 1]$  dimensional multivariate standard normal *cdf* with correlation coefficient matrix as  $R_C$ , and evaluated at vector  $b$ . Vector  $b$  is  $[(M - k + 1) \times 1]$  and can be shown to be equal to  $E \cdot G$ , where  $E$  is a  $[M - k + 1]$  diagonal matrix with diagonal elements equal to

$$((C_1 R_C' C_1)^{-1/2}, (C_{k+1} R_C' C_{k+1})^{-1/2}, \dots, (C_M R_C' C_M)^{-1/2});$$

where

$$C = \begin{pmatrix} C_1 \\ C_{k+1} \\ \vdots \\ C_M \end{pmatrix} = H \cdot D^{\frac{1}{2}},$$

$$H = \begin{bmatrix} \frac{1}{S_1} & 1 & 1 & \dots & 1 \\ 0 & -1 & 0 & \dots & 0 \\ 0 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & -1 \end{bmatrix},$$

a  $[M - k + 1]$  square matrix,  $R$  is the correlation coefficient matrix derived from  $\Omega_{11}$ , and  $D$  the diagonal elements of  $\Omega_{11}$ . Term

$$G = \begin{pmatrix} 1 - H_1 U^* \\ -U_{k+1}^* \\ \vdots \\ -U_M^* \end{pmatrix},$$

<sup>6</sup> The transformation in this Appendix is very similar to Dong et al. (2004). The difference roots at endogenizing unit values. We repeat the transformation procedure here to keep the paper consistent and readable.

where  $H_1$  is the first row of matrix  $H$ . The new correlation coefficient matrix ( $R_C$ ) is given as  $R_C = ECRC'E'$  (Tallis, 1965).

Equation (A.2) represents a rectangular standard multivariate normal probability, which can be conveniently evaluated using standard simulation procedures such as *GHK*. This equation is represented as (23) in the text.

#### Derivation of Equation (24)

Equation (24) is the likelihood for Regime II, in which where the number of commodities actually purchased,  $k$ , equals  $M$ . Under this special case, Eq. (A.2) can be restated as

$$L(S_1, S_2, \dots, S_M > 0; S_{M+1} = 0) \\ = B_1 \cdot \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \cdot \int_{S_1}^{+\infty} \phi(S_1^*, U^*, \Omega_{11}) dS_1^*, \quad (\text{A.3})$$

where  $U^* = U_1^* = \Omega_{11}^{-1} \Omega_{10}^{-1} U_1$ , and  $\Omega_{11} = I' \sigma_{11} I$ ,  $\Omega_{00} = J' \sigma_{11} J$ ,  $\Omega_{10} = I' \sigma_{11} J$  are all scalars now with  $\sigma_{11} =$

$(A \Sigma_{\omega\omega}^{-1} A')^{-1}$ ,  $A$  is an  $M \times M$  diagonal matrix with diagonal elements:

$$\left(1, \frac{S_2}{S_1}, \frac{S_3}{S_1}, \dots, \frac{S_k}{S_1}\right),$$

and  $I$  a  $[M \times 1]$  vector of ones,

$$J = \left(1, \frac{U_2}{\left(\frac{S_2}{S_1}\right) U_1}, \frac{U_3}{\left(\frac{S_3}{S_1}\right) U_1}, \frac{U_4}{\left(\frac{S_4}{S_1}\right) U_1}, \dots, \frac{U_M}{\left(\frac{S_M}{S_1}\right) U_1}\right)',$$

and  $B_1 = (2\pi)^{\frac{1-M}{2}} \cdot |\Sigma_{\omega\omega}|^{-\frac{1}{2}} \cdot |\Omega_{11}|^{\frac{1}{2}} \cdot e^{-\frac{1}{2} \{U_1' \Omega_{00}^{-1} U_1 - U_1' \Omega_{11}^{-1} U_1\}}$ .

Thus, under purchase regime II, the likelihood function requires only the integration of a univariate *PDF*. Equation (A.3) is represented as (24) in the text.

## Appendix B. Maximum likelihood estimates of parameters in share equations

Variable	Share equation (S)							
	(Fish)		(Beef)		(Pork)		(Other meat)	
	Coefficient	Std. Error	Coefficient	Std. Error	Coefficient	Std. Error	Coefficient	Std. Error
CONSTANT	0.6944*	0.2082	2.4174*	0.3189	-1.0713*	0.2318	-1.0404*	0.2557
Unit values								
FISH	0.1219*	0.0331						
BEEF	-0.1756*	0.0363	0.1929*	0.0673				
PORK	0.0211	0.0240	0.0515	0.0410	-0.0500	0.0380		
OTHER MEAT	0.0326	0.0245	-0.0688*	0.0351	-0.0227	0.0300	0.0589	0.0383
Total expenditure and advertising								
TOTEXP	-0.1952*	0.0125	0.0060	0.0206	0.1486*	0.0174	0.0405*	0.0166
ADV_FISH	0.1485*	0.0711	-0.1736	0.1073	-0.0298	0.0647	0.0548	0.0761
ADV_MEAT	0.0439	0.0259	0.0901*	0.0373	-0.0049	0.0241	-0.0480*	0.0212
Household characteristics and regions								
ln (HSIZE)	0.1499*	0.0306	-0.0433	0.0440	-0.0702*	0.0301	-0.0364	0.0276
ln (AGE_HEAD)	0.3171*	0.0382	-0.3992*	0.0587	-0.0408	0.0382	0.1228*	0.0431
NUM_KID	0.0028	0.0219	0.0340	0.0309	-0.0310	0.0195	-0.0059	0.0194
METRO	0.0109	0.0276	-0.0276	0.0397	-0.0583*	0.0257	0.0751*	0.0265
NORTH	-0.1643*	0.0393	0.0066	0.0537	0.0972*	0.0355	0.0604	0.0313
CENTRAL	-0.1063*	0.0335	0.0002	0.0474	0.0947*	0.0287	0.0114	0.0272
WEST	-0.0837*	0.0307	-0.0092	0.0459	0.0918*	0.0305	0.0010	0.0281
OSLO	0.0934*	0.0340	-0.0681	0.0536	-0.0142	0.0374	-0.0110	0.0336
Cholesky decomposition of variances–covariance matrix								
$\sigma_{ee}$ (FISH)	0.4763*	0.0098						
$\sigma_{ee}$ (BEEF)	-0.3701*	0.0194	0.3402*	0.0110				
$\sigma_{ee}$ (PORK)	-0.1047*	0.0156	-0.4284*	0.0176	0.4338*	0.0110		

\*indicates statistically significant at 0.05 level or higher.

**Appendix C. Maximum likelihood estimates of parameters in unit value equation**

Variable	Unit value equation (V)							
	(Fish)		(Beef)		(Pork)		(Other meat)	
	Coefficient	Std. error	Coefficient	Std. error	Coefficient	Std. error	Coefficient	Std. error
CONSTANT	1.0215*	0.2988	1.7124*	0.1596	2.4697*	0.3074	1.1891	0.7695
Income and advertising								
ln (TOTEXP)	0.0700*	0.0279	0.0594*	0.0149	0.0957*	0.0270	0.1024*	0.0508
ADV_FISH	0.0083	0.0871	−0.0725	0.0469	−0.3460*	0.0877	−0.2009	0.2276
ADV_MEAT	0.0168	0.0317	0.0237	0.0168	0.1001*	0.0315	−0.0207	0.0790
Household characteristics and regions								
ln (HSIZE)	−0.0780	0.0466	−0.0295	0.0235	−0.1728*	0.0445	−0.0764	0.1004
ln (AGE_HEAD)	0.0385	0.0541	−0.0721*	0.0286	−0.2605*	0.0561	0.0572	0.1532
NUM_KID	−0.0430	0.0268	−0.0307*	0.0147	0.0043	0.0251	0.0103	0.0676
METRO	0.0153	0.0357	0.0391*	0.0192	0.0229	0.0335	−0.1974*	0.0873
NORTH	−0.0035	0.0489	0.0652*	0.0275	0.0291	0.0593	0.2315*	0.1158
CENTRAL	0.0042	0.0435	0.0108	0.0215	0.0776*	0.0392	0.1451	0.1106
WEST	0.0652	0.0382	0.0298	0.0222	0.0919*	0.0409	0.2442*	0.0964
OSLO	0.0410	0.0463	0.0280	0.0230	0.0751	0.0437	0.0154	0.0971
Cholesky decomposition of variances–covariance matrix								
$\sigma_{\varepsilon\varepsilon}$ (FISH)	0.5363*	0.0074						
$\sigma_{\varepsilon\varepsilon}$ (BEEF)	0.0522*	0.0089	0.3050*	0.0039				
$\sigma_{\varepsilon\varepsilon}$ (PORK)	0.0502*	0.0181	0.0807*	0.0099	0.4779*	0.0098		
$\sigma_{\varepsilon}$ (OTHER MEAT)	0.0584	0.0462	0.0809*	0.0323	0.1390*	0.0403	0.5554*	0.0227

\*indicates statistically significant at 0.05 level or higher.

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